A.2

Midterm Exam of Calculus I (Fall 2000)

Problem 1. (10pts)
Let $S = \{x \in \mathcal{R} |\ x^3 - 6x^2 + 11x - 6 < 0\}$. Find $\sup S$ and $\inf S$.

Problem 2. (10pts) Draw the graph of $y = 2x - \lfloor x \rfloor$.

Problem 3. (10pts)
If $f : \mathcal{R}\backslash\{1\} \longrightarrow \mathcal{R}$ such that $f(x) = \frac{x+1}{x-1}$. Find $f^{-1}(y)$.

Problem 4. (20pts)
(a) Write down the definition of $\lim_{x \to b} f(x) = B$.
(b) Prove that $\lim_{x \to 2} \sqrt{x} = \sqrt{2}$ using $\epsilon - \delta$ method.

Problem 5. (10pts) Find the following limits and show your work:
(a) $\lim_{x \to 2} x \lfloor \frac{1}{x} \rfloor$.
(b) $\lim_{x \to 0} x \lfloor \frac{1}{x} \rfloor$.

Problem 6. (20pts) Find the following limits and show your work:
(a) $\lim_{x \to \infty} \sqrt{x+1} - \sqrt{x}$,
(b) $\lim_{x \to 1} \frac{x+x^2+x^3+x^4+x^5+x^6-6}{x-1}$,
(c) $\lim_{x \to \infty} \frac{(x-4)^{15}(2x+1)^{25}}{(4x+1)^{40}}$,
(d) $\lim_{x \to 0^-} \frac{(x\lfloor x \rfloor)}{|x|}$.

Problem 7. (10pts) Suppose that we have $\lim_{x \to \infty} (\frac{2x^2}{x+1} - ax - b) = 2$. Find the constants a and b.

Problem 8. (20pts) Find the following limits and show your work:
(a) $\lim_{x \to 0} \frac{\sin x}{x}$,
(b) $\lim_{x \to 0} \frac{2-2\cos^2(\frac{x}{2})}{x}$,
(c) $\lim_{x \to 0} \frac{(\sin x)(1-\cos x)}{x^3 \cos x}$,
(d) $\lim_{x \to 0} \frac{(\tan x)(1-\cos x)}{x^2 \tan(3x)}$.